COBEM 99
XV Congresso Brasileiro de Engenharia Mecânica
15th Brazilian Congress of Mechanical Engineering
22 - 26 de Novembro de 1999 / November 22 - 26, 1999 Águas de Lindóia, São Paulo,

MODELING OF A CLOSED-LOOP CHAIN MANIPULATOR BY THE RECURSIVE NEWTON-EULER METHOD

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Abstract. *In this article, a method is proposed to obtain the equations of motion of a pantographic closed-loop chain manipulator, with three degrees of freedom, using the Newton-Euler formulation. The mathematical model is developed dividing the mechanism in open-loop chains while preserving the constraint relations due to the pantographic mounting. In each of the open-loop chains, an external force applied in its extremity may be determined by means of a vectorial equation system so that the torques in the passive joints axis are null. These joints are the ones where there is no actuator providing external torque. After these forces are determined, they enter as reaction forces in another open-loop chain, determining the solution of the inverse dynamics problem. Through the adequate manipulation of some input vectors of the inverse dynamics procedure, it is possible to obtain the mass matrix of the system and the vectors related with the action of the gravity, Coriolis and centrifuge inertia forces, setting up the direct dynamic system. Results of simulations are presented for some trajectories. The method, with small modifications, can be adapted to other closed-loop chain mechanisms as long as the constraints relating the passive to the active joints are established.*

Keywords: *Robotics, Closed-loop chain, Inverse dynamics modeling*

1. INTRODUCTION

A robotic manipulator may be defined as an electro-mechanic device that has the function of positioning and orientating a mechanism located in its extremity, the hand, as described by Martins *et al.* (1991). The main function of this mechanism is to permit the fixation of tools or other devices, that depend on the type of task to be performed. Two parts should be considered in a manipulator structure. The first is the arm, which consists of at least three degrees of freedom (dofs) and used to position the hand. The second part is the wrist, usually constituted by other three rotational dofs for hand orientation.

However, for the accomplishment of specific tasks, it is not necessary that the hand has complete orientation freedom around a point. In order to reduce its cost, robots dedicated to perform certain tasks have been proposed. These structures usually have less than five dofs and a configuration such that its hand possesses the desired freedom of movement.

1.1 Mechanical structure

To present the method, a pantographic mechanism, similar to the one proposed by Zampieri *et al.* (1991) and illustrated in fig. 1, was considered. The structure has eleven kinematic pairs but 4 generalized coordinates are sufficient to describe its motion, the first one being the rotation at the waist. The fourth generalized coordinate is dependent on the values of the previous two, which is due to the pantographic characteristic of the mechanism, giving to the wrist the property of rotating only around the axis of rotation of the waist (joint 1). This feature makes this type of structure adequate to perform tasks that need this attribute at the wrist, as for example painting, welding, machining etc. Another important advantage of this structure is the high rigidity given by the closed chain and the good accuracy of orientation of the hand terminal, independent of the control system. External loadings on the manipulator wrist are not considered in the following presentation.

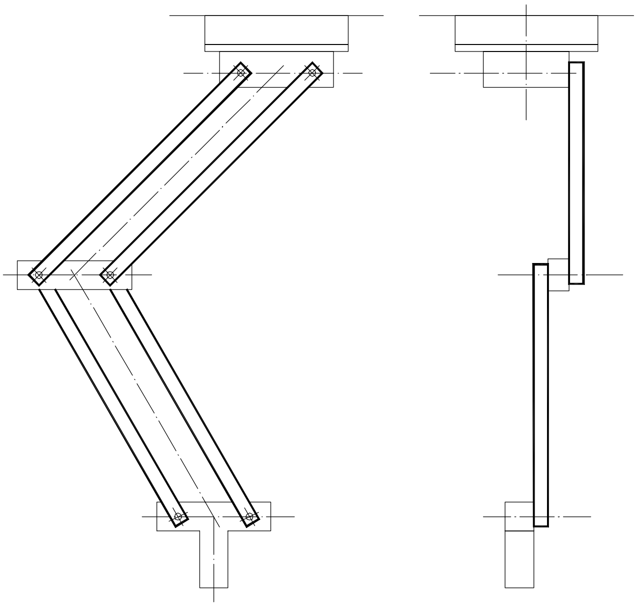

Figure 1 – Representation of the pantographic mechanism.

2. FORMULATION OF THE INVERSE DYNAMICS

The objective of the inverse dynamics is to determine the driving forces required by the actuators in order to perform a given trajectory. To do so, a procedure proposed initially by Luh *et al.* (1980), based on the Newton and Euler equations applied to rigid bodies, is usually employed.

In this procedure, after the characterization of the parameters of inertia of the manipulator and the kinematics of the problem in the joint space, the values of angular velocity and angular and linear acceleration at the center of mass of each link are calculated in a recursive form, beginning at the base and finalizing at the wrist. Next, after the determination of the resultants of the forces and moments around the center of mass of each link, the required torques are established through the determination of the forces and moments acting at the joints at each link, also in a recursive form, beginning now at the wrist and concluding at the base (Craig, 1989, and Yoshikawa, 1990). However, as implicit in the algorithm description, its application is only described for open chain manipulators and with external loading only at its free end.

Analyzing the pantographic mechanism represented in fig. 1, it may be observed that it can be divided into two open chains that interact one with the other, as illustrated in figures 2 and 3. In these figures, numbers are used to represent active joints (with actuators) and links connected to these. Letters are used to represent passive joints and links not connected to active joints.

The first mechanism represents the last quadrilateral and its kinematic connection with the base. The objective now is to determine the force and the moment that acts at its free end, point N, and produces no binaries in the Z direction of L and M joints. The determination of these allow the use of the usual Newton-Euler procedure to determine the required torque in joint three. It is evident that being a methodology based on rigid body mechanics, some components of the force and moment vectors are undetermined. More specifically, the Z component of the force and the X and Y components of the moment. However, the determination of the torque in joint 3, that is, of component Z of the moment N_3, does not require these undetermined values. A symbolic analysis, which was undertaken to derive the equations presented here, using MATLAB (1997), confirms this assertive.

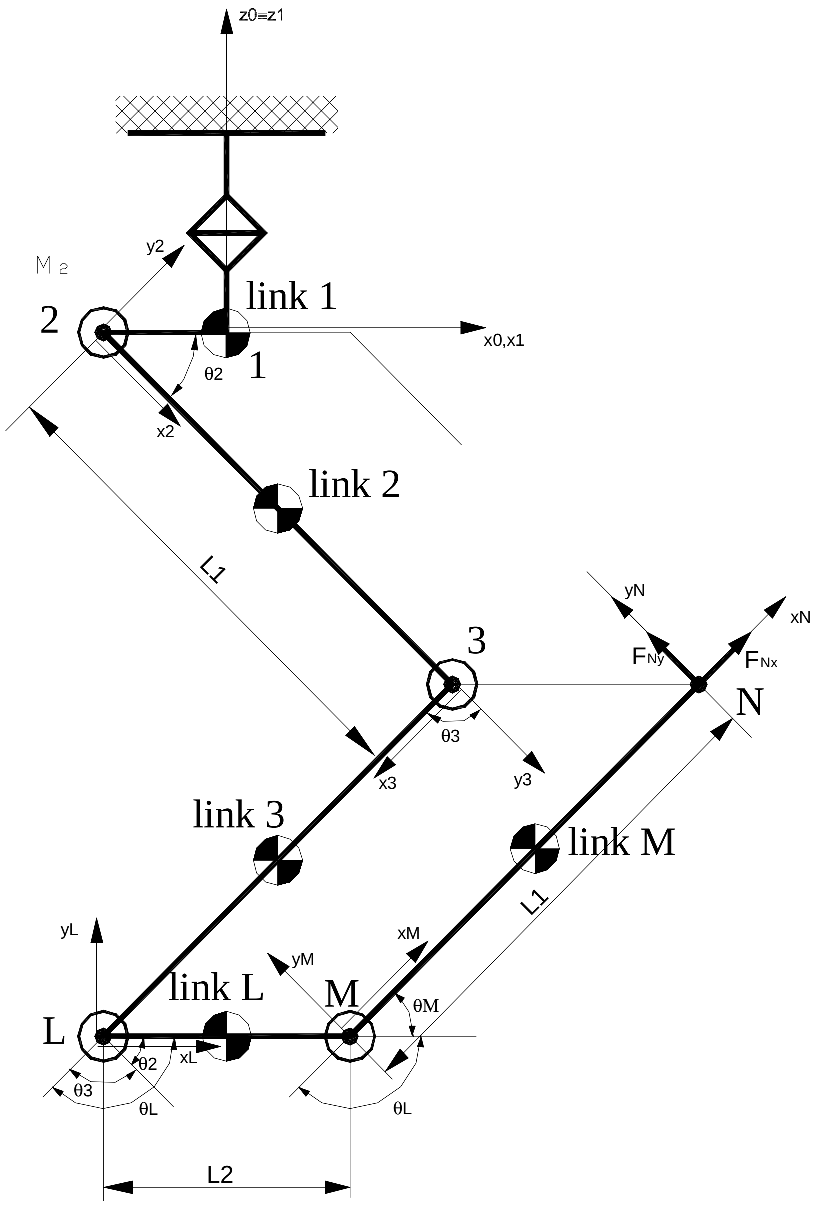

Figure 2 – Frame positions for the first mechanism.

The expressions to calculate the forces and moments at joints L, M and 3 are:

$$F_M = R_{NM} \cdot F_N + Fc_M \tag{1}$$

$$N_M = R_{NM} \cdot N_N + Nc_M + s_M \times Fc_M + p_{NM} \times \left(R_{NM} \cdot F_N\right) \tag{2}$$

$$F_L = R_{ML} \cdot F_M + Fc_L \tag{3}$$

$$N_L = R_{ML} \cdot N_M + Nc_L + s_L \times Fc_L + p_{ML} \times \left(R_{ML} \cdot F_M\right) \tag{4}$$

$$F_3 = R_{L3} \cdot F_L + Fc_3 \tag{5}$$

$$N_3 = R_{L3} \cdot N_L + Nc_3 + s_3 \times Fc_3 + p_{L3} \times \left(R_{L3} \cdot F_L\right) \tag{6}$$

where:

F_i is the force at joint *i*
N_i is the moment at joint *i*
$R_{i,i-1}$ is the rotation matrix of frame *i* relative to frame *i-1*
$p_{i,i-1}$ is the position vector of frame *i* relative to frame *i-1*
s_i is the position vector of the center of mass of link *i*, relative to its frame
Fc_i is the resultant force at link *i*
Nc_i is the resultant moment at link *i*, in relation to the center of mass

The Denavit-Hatenberg (1955) parameters for the first mechanism, in accordance with the frames presented in fig. 2, are presented in table 1 together with the moments of inertia of it's links. From the former, the constraint equations may be expressed as:

$$\begin{aligned} \theta_L &= -\theta_2 - \theta_3 & \qquad \theta_M &= 180^\circ - \theta_L = 180^\circ + \theta_2 + \theta_3 \\ \dot{\theta}_L &= -\dot{\theta}_2 - \dot{\theta}_3 & \qquad \dot{\theta}_M &= \dot{\theta}_2 + \dot{\theta}_3 \\ \ddot{\theta}_L &= -\ddot{\theta}_2 - \ddot{\theta}_3 & \qquad \ddot{\theta}_M &= \ddot{\theta}_2 + \ddot{\theta}_3 \end{aligned} \tag{7}$$

Table 1 – Denavit-Hartenberg and inertia parameters for the first mechanism

i	α_{i-1}	a_{i-1}	θ_{i-1}	d_{i-1}	m_i	r_x	r_y	r_z	I_{xx}	I_{yy}	I_{zz}
1	0	0	θ_1	0	m_1	0	0	$\frac{L_0}{2}$	$I_{1_{XX}}$	$I_{1_{YY}}$	$I_{1_{YY}}$
2	90	$\frac{-L_2}{2}$	θ_2	0	m_2	$\frac{L_1}{2}$	0	0	0	$I_{2_{YY}}$	$I_{2_{YY}}$
3	0	L_1	θ_3	0	m_2	$\frac{L_1}{2}$	0	0	0	$I_{2_{YY}}$	$I_{2_{YY}}$
L	0	L_1	$-\theta_2 - \theta_3$	0	m_3	$\frac{L_2}{2}$	0	0	0	$I_{3_{YY}}$	$I_{3_{YY}}$
M	0	L_2	$180 + \theta_2 + \theta_3$	0	m_2	$\frac{L_1}{2}$	0	0	0	$I_{2_{YY}}$	$I_{2_{YY}}$
N	0	L_1	0	0	0	0	0	0	0	0	0

Solving equation (2) for null value for the Z component of N_M, since there is no external torque, an expression for the Y component of the external force applied at the free end of the mechanism is derived:

$$F_N(y) = \frac{-Fi_M(z)}{L_1} \tag{8}$$

where $Fi_M(z)$ is the third component of $(Nc_M + s_M \times Fc_M)$ and whose value may be determined through the first recursive loop of the Newton-Euler algorithm. In an analogous way, solving equation (4) for null Z component of N_L, an expression for the X component is obtained:

$$F_N(x) = \frac{Fi_L(z) - L_2 \cdot sin(q_2+q_3) \cdot Fc_M(x) + \dfrac{L_2 \cdot cos(q_2+q_3) \cdot Fi_M(z)}{L_1} - L_2 \cdot cos(q_2+q_3) \cdot Fc_M(y)}{L_2 \cdot sin(q_2+q_3) \cdot Fc_M(x)} \tag{9}$$

where $Fi_L(z)$ is the Z component of $(Nc_L + s_L \times Fc_L)$ and $Fc_M(x)$ and $Fc_M(y)$ are the X and Y components of the vector Fc_M, which is also calculated through the first loop. The variables q_2 and q_3 are the second and third joint displacements, respectively. The Z component of the moment at joint 3 may now be calculated from the following expression:

$$\tau_3(z) = Fi_3(z) + Fi_M(z) - Fc_M(y) \cdot L_1 - Fc_L(x) \cdot L_1 \cdot sin(q_2+q_3) + cos(q_2+q_3) \cdot L_1 \cdot Fc_L(y) \tag{10}$$

where $Fi_3(z)$ is the Z component of $(Nc_3 + s_3 \times Fc_3)$.

Therefore, it is possible to derive an analytical expression for the required torque at joint 3, which is one of the unknowns of the inverse dynamics problem. To introduce external loading on the manipulator wrist, it would be enough to introduce additional terms in equations 3 and 4, similar to the ones employed in the following equations 11 and 12 or 15 and 16 or 17 and 18.

The function of the second mechanism is to determine the binaries in joints 1 and 2. The force and moment equations of this system have to consider the reaction forces caused by the connections with the previous structure. The modified expressions, in order to contemplate the reaction forces, are:

$$F_B = R_{DB} \cdot F_D + Fc_B + R_{NB} \cdot (-F_N) \tag{11}$$

$$N_B = R_{DB} \cdot N_D + Nc_B + s_B \times Fc_B + p_{DB} \times (R_{DB} \cdot F_D) + R_{NB} \cdot (-N_N) + p_{NB} \times (R_{NB} \cdot (-F_N)) \tag{12}$$

$$F_A = R_{BA} \cdot F_B + Fc_A \tag{13}$$

$$N_A = R_{BA} \cdot N_B + Nc_A + s_A \times Fc_A + p_{BA} \times (R_{BA} \cdot F_B) \tag{14}$$

$$F_2 = R_{A2} \cdot F_A + Fc_2 + R_{K2} \cdot F_K \tag{15}$$

$$N_2 = R_{A2} \cdot N_A + Nc_2 + s_2 \times Fc_2 + p_{A2} \times (R_{A2} \cdot F_A) + R_{K2} \cdot N_K + p_{K2} \times (R_{K2} \cdot F_K) \tag{16}$$

$$F_1 = R_{21} \cdot F_2 + Fc_1 + R_{01} \cdot (-F_D) \tag{17}$$

$$N_1 = R_{21} \cdot N_2 + Nc_1 + s_1 \times Fc_1 + p_{21} \times \left(R_{21} \cdot F_2\right) + R_{01} \cdot \left(-N_D\right) + p_{01} \times \left(R_{01} \cdot \left(-F_D\right)\right) \quad (18)$$

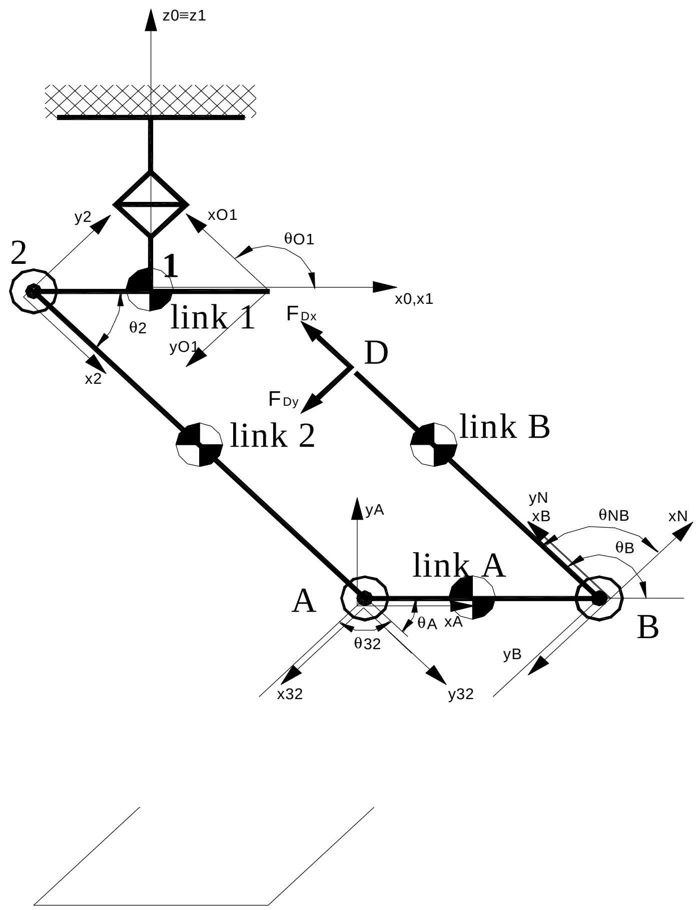

Figure 3 – Frame positions for the second mechanism.

The constraint equations for the second mechanism are:

$$\begin{array}{lllll}
\theta_B = \pi + \theta_2 & \theta_{C1} = \pi + \theta_2 & \theta_{K2} = \theta_2 & \theta_A = -\theta_2 & \theta_{NB} = \theta_2 \\
\dot{\theta}_B = \dot{\theta}_2 & \dot{\theta}_{C1} = \dot{\theta}_2 & \dot{\theta}_{K2} = \dot{\theta}_2 & \dot{\theta}_A = -\dot{\theta}_2 & \dot{\theta}_{NB} = \dot{\theta}_2 \\
\ddot{\theta}_B = \ddot{\theta}_2 & \ddot{\theta}_{C1} = \ddot{\theta}_2 & \ddot{\theta}_{K2} = \ddot{\theta}_2 & \ddot{\theta}_A = -\ddot{\theta}_2 & \ddot{\theta}_{NB} = \ddot{\theta}_2
\end{array} \quad (16)$$

For the second mechanism, the torque at joint 2 is function of $F_D(x)$ and $F_D(y)$ as well as of $F_N(x)$ and $F_N(y)$, which were previously calculated. Solving equations (12) and (14) for null values for the Z components of N_B and N_A, the values of F_{Dx} and F_{Dy} are determined. These values are substituted into to the Z components of N_2 and N_1, in a similar way as in the first mechanism, determining the solution of the complete inverse dynamics problem.

It is important to note that the procedure presented in this section may be said to be a hybrid one. That is, the first recursive loop is performed numerically while an analytical expression, established by a symbolic analysis, was used instead of the second recursive loop. This is considered to be a reasonably efficient technique, leading to smaller computational effort and less numerical error (Machado, 1996).

3. SIMULATIONS

In order to test the mathematical model obtained by the procedure presented in the previous section, two simulations were made with MATLAB (1997). The first simulation performed two direct dynamics analysis, mounted by means of the Walker & Orin algorithm (1980). For the numeric solution of a direct dynamics problem by means of a step by step integration, this algorithm employs the inverse dynamics in order to determine the mass matrix and the vectors related to the gravity, Coriolis and centrifuge inertia forces for each integration step, through the manipulation of the input vectors of angular position, velocity and acceleration in the joint space.

The initial values of the joint displacement and velocity vectors of the first analysis were:

$$q = \begin{bmatrix} 0 & -70^0 & 0 \end{bmatrix}^T \quad \text{and} \quad \dot{q} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

While the initial values of the second analysis were:

$$q = \begin{bmatrix} 0 & 0 & 50^0 \end{bmatrix}^T \quad \text{and} \quad \dot{q} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

The results are shown in figures 4 and 5, representing free oscillations from the specified initial conditions under self weight loading in the –Z direction of the inertial base frame, see figure 2. Therefore, in the two simulations, joint two oscillates around –90 degrees and the other two joints around zero degrees, which are their equilibrium positions. In these analyses, only the coherence of the results were verified.

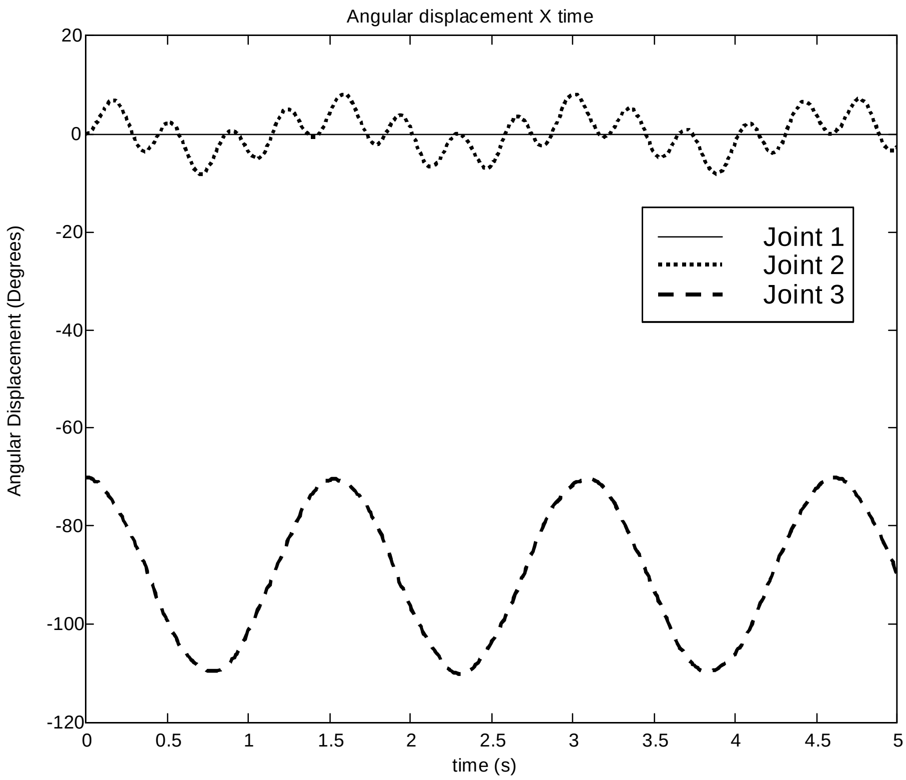

Figure 4 – Results of the first analysis.

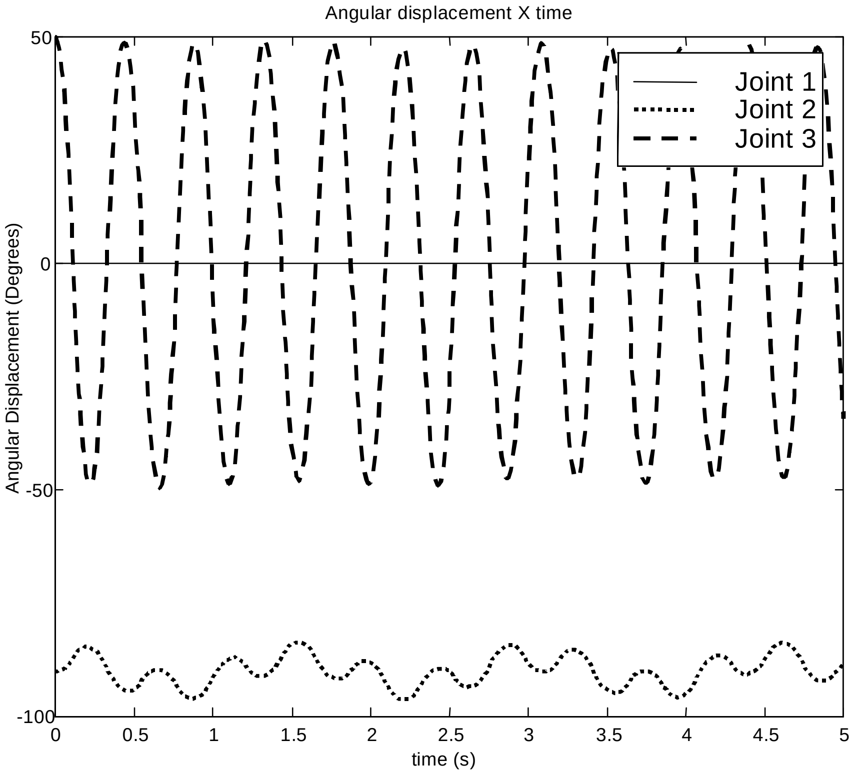

Figure 5 – Results of the second analysis.

The second simulation calculates the required driving forces for a specified trajectory directly from the mathematical model. The trajectory was represented by means of a fifth degree polynomial, where the initial and final vectors of joint coordinates were:

$$q_i = \begin{bmatrix} 0 & -90^0 & 0 \end{bmatrix}^T \quad \text{and} \quad q_f = \begin{bmatrix} 90^o & -45^0 & -18^0 \end{bmatrix}^T$$

and the time at the end of the trajectory is 5.0s. The trajectories and required torques are presented in fig. 6. The torques at the initial and final positions, where the velocity and the acceleration are specified to be null, were compared with the statically calculated values at these positions and agreed.

4. CONCLUSION

In order to solve for the inverse dynamics problem of a closed-loop chain manipulator, a procedure, based on the recursive Newton-Euler algorithm was developed. The procedure employs a hybrid approach to solve the problem. Simulations show that the results are coherent. The method, with small modifications, can be adapted to other closed-loop chain mechanisms.

Once established the simulation procedure, design of the manipulator and its control follows. Afterwards, a prototype should be mounted and tested.

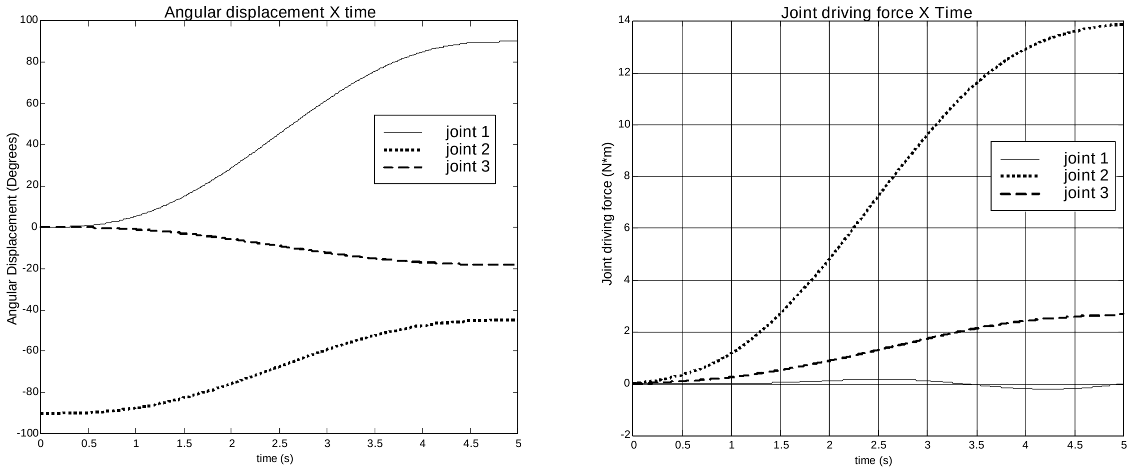

Figure 6 – Input and results from the inverse dynamics simulation

Acknowledgemnets

The authors are grateful to CNPq for the grants awarded to two of them.

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